TRACING CLOSED CURVES WITH EPICYCLES: A FUN APPLICATION OF THE DISCRETE FOURIER TRANSFORM

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Abstract

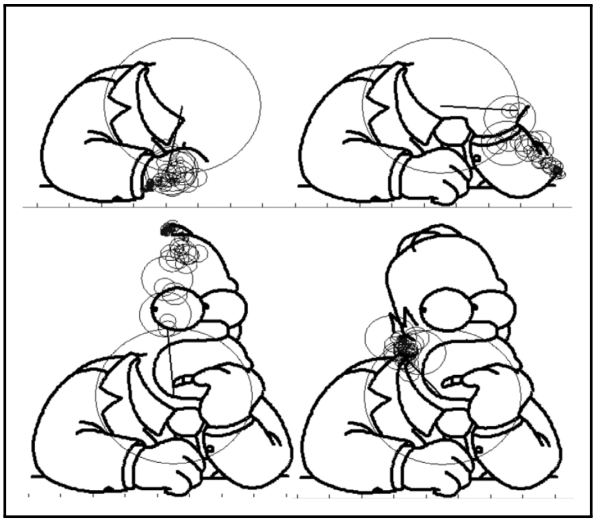
The Discrete Fourier Transform has many applications in our modern digital world. In particular, it allows us to approximate periodic functions by means of trigonometric polynomials which provides the required information to define a system of epicycles that can be animated to trace out closed curves. In this paper, I present a method in GeoGebra to create artistic animations consisting of systems of epicycles tracing out closed curves. The geometric construction presented here can also be used as an introductory learning activity to study the Discrete Fourier Transform from a geometric point of view.

Keywords: trigonometric, interpolation, Discrete Fourier Transform, scripting, epicycles

1 INTRODUCTION

In 2008 Santiago Ginnobili y Christián C. Carman, inspired by the work of Hanson (1965) decades earlier, published an example of a system of epicycles tracing out an orbit resembling the famous cartoon character, Homer Simpson (Ginnobili and Carman (2008)).

Figure 1. Homer Simpson's orbit with 1000 epicycles by Ginnobili and Carman (2008).



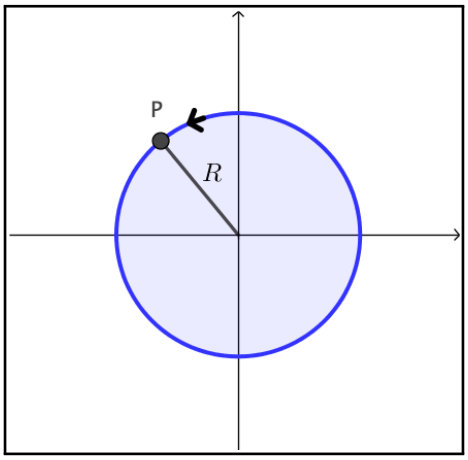
In that same year, Ginnobili published an online video with an animated version of this orbit in which we can observe an assemblage of 1000 circles with different radii, each one of them rotating at different speeds and slowly tracing out Homer Simpson's orbit, see Figure 1. To fully appreciate how striking this animation is, I highly recommend the reader to watch the video by Ginnobili (2008). This intricate animation relies on the deep insights arising from the Fourier Transform. More specifically on the Discrete Fourier Transform.

How did Ginnobili and Carman manage to make this animation? How can we calculate the radii of the circles? How can we calculate the speed at which each circle rotates? But most importantly, how is this animation related to the Discrete Fourier Transform? In this paper I will discuss the mathematics required to answer all these questions and I will provide a method to create in GeoGebra artistic animations consisting of systems of epicycles tracing out closed curves.

2 WHAT ARE EPICYCLES?

Consider a single point P on the edge of a rotating disk. If we traced it out for a full cycle, that point describes a circle:

Figure 2. The point P with an anticlockwise rotation describes a circle.



We can represent the motion of this point P considering the center of the circle as the origin of the Cartesian coordinate system and using parametric equations:

$$\begin{aligned} x(t) &= R\cos(\omega t) \\ y(t) &= R\sin(\omega t); \end{aligned}$$

where R is the radius of the disk and ω is the speed at which the disk is rotating. If ω is positive, the rotation is anticlockwise, and if it is negative, the rotation is clockwise.

Now let's look at something a bit more complex: Let's add another disk with radius half of the original disk and centred at P (point rotating on the original disk) but this time rotating clockwise (three times faster) with its own point Q rotating on its circumference. If we trace the locus of this second point, we can observe a shape resembling a flower, shown in Figure 3.